Tunable magnetoelastic phononic crystals

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The feasibility of tuning the band structure of phononic crystals is demonstrated by employing magnetostrictive materials and applying an external magnetic field. Band structures are calculated with a plane wave expansion method that accounts for coupling between the elastic behavior and the magnetic field through the development of elastic, piezomagnetic, and magnetic permeability effective tensors. We show the contactless tunability of the absolute band gaps of a two-dimensional phononic crystal composed of an epoxy matrix and Terfenol-D inclusions. The tunable phononic crystal behaves like a transmission switch for elastic waves when the magnitude of an applied magnetic field crosses a threshold. © *2009 American Institute of Physics*. [doi:10.1063/1.3236537]

Phononic crystals are composite structures made of periodic arrays of elastic inclusions embedded in an elastic matrix. By varying the geometry and constitutive materials, one can introduce a variety of features in the elastic band structures of the phononic crystals such as absolute band gaps[1] or passing bands with negative group velocity.[2] Therefore phononic crystals can be used in devices with potentially useful properties including filters,[3] waveguides,[4] and lenses.[5] One of the major stumbling block to the application of phononic crystals is the lack of practical frequency tunability of their properties. Tunability could be achieved by changing the geometry of the inclusions[6] or by varying the elastic characteristics of the constitutive materials through application of external stimuli.[7] For instance, some authors have proposed the use of electrorheological materials in conjunction with application of external electric field.[8] Other authors have considered the effect of temperature on the elastic moduli.[9] In all cases, significant effect on the band structure of the phononic crystal can only be achieved by applying stimuli with very large magnitude. Recent work[10] exploits the change of the structure of the phononic crystal due to an external stress to alter the band structure. However this approach requires physical contact with the phononic crystal.

We propose to tune the properties of phononic crystals using magnetoelastic components. The elastic properties of magnetoelastic material are very sensitive to its magnetic state and on the applied magnetic field. For instance, in giant magnetostrictive material, such as Terfenol-D, this dependence can lead to more than 50% variation of some of the elastic constants, even at ultrasonic frequencies.[11] Several studies have reported noticeable changes in the band structures of magnetoelectroelastic phononic crystals when the coupling between magnetic, electric, and elastic phenomena are taken into account.[12,13] These studies however do not consider the effect of an external magnetic field on the properties of the phononic crystal. In the present letter, we demonstrate that the band structure of a two-dimensional (2D) phononic crystal constituted of a magnetoelastic medium can be controlled by application of an external magnetic field. This approach offers an effective means of contact-less tunability of the properties of phononic crystals.

It has been demonstrated that if the magnetic dipole interaction is neglected, the elastic dynamics of a homogeneous magnetoelastic medium can be described in terms of effective elastic moduli.[14] Taking into account the magnetic dipole interaction in the case of elastic plane wave excitations leads to effective elastic moduli that depend on the direction of propagation.[15] This dependence makes the use of effective elastic moduli unpractical in band structure calculation methods such as plane wave expansion (PWE) or finite element (FE) method. To circumvent the problem of the calculation of the band structure of a magnetoelastic phononic crystal, we derive the equivalent piezomagnetic material of a polarized ferromagnet. In this case, the elastic constants C_{ijkl}, piezomagnetic constants q_{lij}, and magnetic permeability μ_{ij} are only field dependent and are readily compatible with standard PWE and FE methods.

We consider a magnetoelastic wave in a ferromagnet magnetized to saturation. In this case, the amplitude of the magnetization $\mathbf{M}$ is a constant M_s. The coupled equations for the mechanical and magnetic systems, i.e., the equation of motion and the Landau–Lifshitz equation, have the form[16] $\rho\ddot{u}_i=\sigma_{ij,j}$, and $\dot{\mathbf{M}}=-\gamma\mathbf{M}\times\mathbf{H}_{\text{eff}}$, where ρ is the density, γ is the gyromagnetic ratio, and u_i are the components of the particle displacement. Here and in the following, the summation over repeated indices is employed. The effective magnetic field $\mathbf{H}_{\text{eff}}=\mathbf{H}-\partial U/\partial\mathbf{M}$ and the stress tensor $\sigma=\partial U/\partial\mathbf{E}$, where $\mathbf{E}$ is the strain tensor, are derived from the internal energy density U of the ferromagnet. As we are interested in acoustic wave propagation, we consider only small dynamic perturbations around an equilibrium state: $\mathbf{H}=\mathbf{H}^0+\mathbf{h}$, $\mathbf{M}=\mathbf{M}^0+\mathbf{m}$, and $\mathbf{E}=\mathbf{E}^0+\varepsilon$. $\mathbf{H}^0$ is the internal field, i.e., the sum of the external applied field $\mathbf{H}^e$ and a demagnetizing field, $\mathbf{E}^0$ is a spontaneous strain caused by magnetostriction, and $\mathbf{h}$, $\mathbf{m}$, and ε are the magnetostatic field, magnetization, and strain generated by the magnetoelastic waves, respectively.

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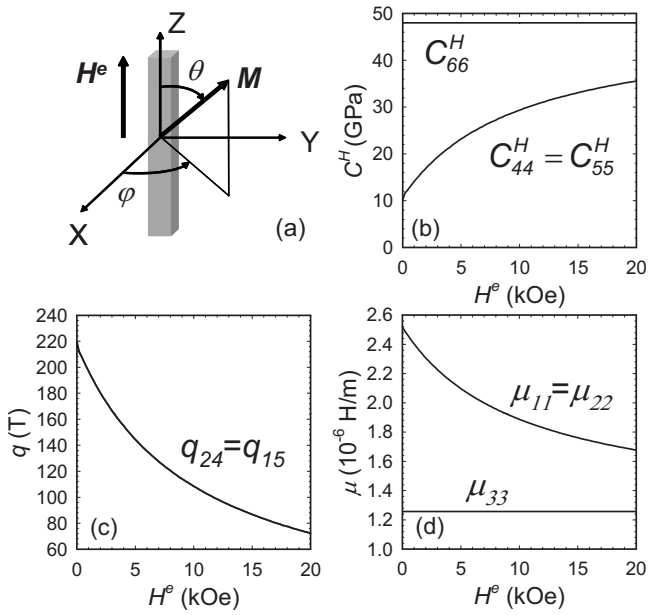

FIG. 1. (a) Schematic illustration showing the spherical coordinates used in the calculation and evolution of the effective, (b) elastic moduli, (c) piezomagnetic constants, and (d) magnetic permeabilities of a Terfenol-D rod as a function of the applied static external magnetic field. The effective elastic and piezomagnetic constants are expressed in Voigt notation.

To be able to consider an external static magnetic field in an arbitrary direction, we use the method of spherical coordinates proposed for the study of ferromagnetic resonance in anisotropic media.[17,18] In this case, as the considered ferromagnet is magnetized to saturation, the two variables, θ and φ as shown on Fig. 1(a), replace the three M_i components. First, the equilibrium state $\mathbf{M}^0$, corresponding to θ_0, φ_0, and $\mathbf{E}^0$, is found by minimizing the total energy of the magnetoelastic system $U_T=U-\mu_0\mathbf{M}\cdot\mathbf{H}^0$, where μ_0 is the magnetic permeability of vacuum. The acoustic perturbation rotates the magnetization to a new θ and φ orientation, where $\theta=\theta_0+\delta\theta$ and $\varphi=\varphi_0+\delta\varphi$. In the frequency range of elastic wave, $\omega \ll \omega_S$ where ω_S is the natural frequency of spin mode, the magnetic subsystem has time to adjust itself to the elastic subsystem, leading to the following conditions $\partial U_T/\partial(\delta\theta)=0$ and $\partial U_T/\partial(\delta\varphi)=0$. Expending the energy around the equilibrium position,

$$U=(U_{\theta\theta}/2)\delta\theta^2+(U_{\varphi\varphi}/2)\delta\varphi^2+U_{\theta\varphi}\delta\theta\delta\varphi+U_{\theta\varepsilon_{ij}}\delta\theta\varepsilon_{ij}+U_{\varphi\varepsilon_{ij}}\delta\varphi\varepsilon_{ij},$$

these conditions enable us to write $\delta\theta$, $\delta\varphi$, and U, as functions of ε and $\mathbf{h}$. U_{xy} denotes the second order x-y derivative of U. Finally, introducing these last expressions in the effective magnetic field and the stress tensor, we can derive the following constitutive equations of an equivalent piezomagnetic material,

$$\sigma_{ij}=C^H_{ijkl}\varepsilon_{kl}-q_{lij}h_l,$$

$$b_i=q_{ikl}\varepsilon_{kl}+\mu_{il}h_l, \tag{1}$$

where b_i are the components of the magnetic induction, μ_{ij} the effective magnetic permeability matrix, q_{ijk} the effective piezomagnetic tensor, and C^H_{ijkl} are the effective elastic constants. These effective material properties are defined as $\mu_{il}=\mu_0\delta_{il}+\chi_{il}$, $q_{mij}=-b_{ijkl}(M^0_k\chi_{lm}+M^0_l\chi_{km})/M^2_S$, and $\Delta C_{ijkl}=C^H_{ijkl}-C_{ijkl}=b_{ijmn}(M^0_n q_{mkl}+M^0_m q_{nkl})/M^2_S$, respectively. C_{ijkl} are the elastic coefficients in absence of magnetic effects. b_{ijkl} are the components of the magnetoelastic coupling tensor. The elements of the susceptibility symmetric tensor χ_{ij} depend on the equilibrium angles θ_0 and φ_0. They are derived from the same calculation.

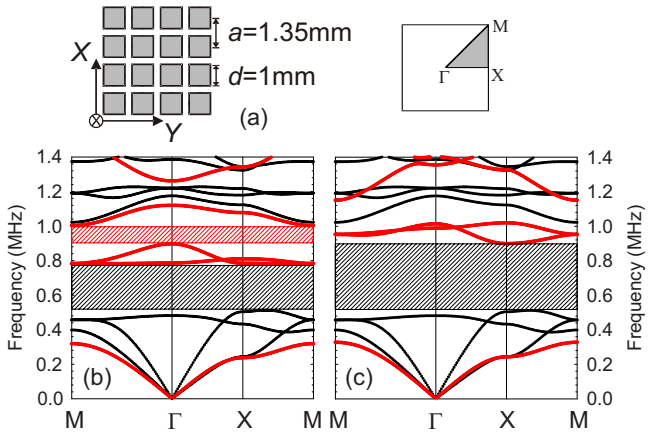

FIG. 2. (Color online) (a) The square-lattice 2D phononic crystal consisting of magneto-elastic square rod of infinite length along the Z direction made of Terfenol-D embedded in an epoxy matrix. Band structure of a square lattice of Terfenol-D square rods with a filling factor $f=(d/a)^2=0.55$, embedded in an epoxy matrix for two applied static magnetic field: (b) $H_{\text{ext}}=0$ kOe and (c) $H_{\text{ext}}=13$ kOe. The inset shows the irreducible Brillouin zone of the square array.

Figures 1(b)–1(d) illustrate the variations of the effective elastic constants, the piezomagnetic coefficients, and magnetic permeabilities of a Terfenol-D rod-shaped ferromagnet as a function of the magnitude of an external magnetic field parallel to the axis of the rod. The chosen Terfenol-D parameters, of cubic symmetry, correspond to the ones of commercially available samples: density $\rho=9210$ kg/m^3, magnetization $M_s=800$ kA/m, magnetic anisotropy constant $K_1=4\times10^5$ J/m^3, magneto-elastic constants $B_1=b_{1111}=-2\times10^7$ J/m^3 and $B_2=b_{1212}=-3.456\times10^8$ J/m^3, and the elastic constants $C_{11}=82$ GPa, $C_{12}=40$ GPa, and $C_{44}=48$ GPa. Since the applied external field is parallel to the Z direction, Fig. 1(b) shows that only the two components $C_{44}=C_{55}$ depend on the magnitude of the external field. The only two nonzero components q_{24} and q_{15} of the effective piezomagnetic tensor are also strongly depend on the external applied magnetic field. Finally the variations of the elements of the diagonal effective magnetic susceptibility matrix are reported in Fig. 1(d).

We used the PWE method for calculating the band structures of 2D arrays of Terfenol-D rods embedded in an epoxy matrix [Fig. 2(a)].[19] One makes use of the quasistatic approximation[12] and develops the equations of propagation in the Fourier space. This leads to a generalized eigenvalue problem where a set of eigenfrequencies are computed for each value of the wave vector of an incident elastic wave. 441 plane waves were sufficient for insuring the convergence of the Fourier series, as verified by comparison with finite elements simulation. We study the behavior of the band structure as a function of the external magnetic field through the variations of the effective characteristics of Terfenol-D shown in Figs. 1(b)–1(d). The band structures of Figs. 2(b) and 2(c) illustrate the effect of the external magnetic field. In absence of an external stimulus, the phononic crystal possesses two absolute band gaps ranging from approximately

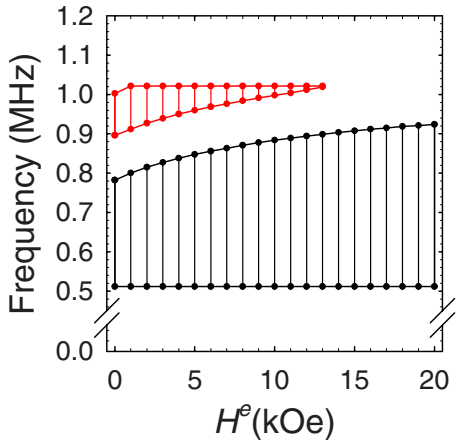

FIG. 3. (Color online) Evolution of the absolute elastic band gaps as a function of the applied static external magnetic field.

0.5 to 0.8 MHz and from 0.89 to 1 MHz. Application of a magnetic field parallel to the Terfenol-D rods with a magnitude of 13 kOe enlarges the first band from 0.5 to 0.92 MHz and closes the second band gap. Contrasting the two band structures one notes that some bands, are not affected by the magnetic field while others, colored in red, are shifted toward higher frequencies. The bands that are altered by the field correspond to modes of vibrations with a polarization parallel to the principal axis of the inclusions (*Z*-axis). This is easily understandable since the magnetic field only influences the shear components of the elastic coefficients C_{44}.[1] The unaffected modes have a polarization lying in the plane perpendicular to the Terfenol-D rods. We summarize the tunability of the band gaps by applying an external magnetic field in Fig. 3. Here we report the widths and edges of the two band gaps as a function of the magnitude of the applied magnetic field. For instance an elastic wave with a frequency of 1 MHz does not propagate in the phononic crystal subjected to a magnetic field less than approximately 12 kOe. When the stimulus exceeds this value, this mode propagates. This phononic crystal behaves like a tunable filter with switching functionality.

We have shown that the band structures of phononic crystals composed of magnetoelastic constituents can be tuned by application of an external magnetic field. The introduction of a magnetoelastic constituent opens the possibility of easy controllability of the properties of a phononic crystal without any contact. More specifically one can achieve additional functionalities such as the switching of transmission in a defined frequency range. This frequency range can be adjusted by varying the geometrical characteristics of the phononic crystals such as the arrangement of inclusions and/or the size and the cross section of the cylindrical inclusions. It is worth noting that the use of giant magnetostriction is not the only means to obtain large elastic properties variation in magnetic materials. Indeed, one can also consider ferromagnetic resonance[15] or spin-reorientation phase transitions[20] effects.

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